4.10   Acceleration

When the velocity of an object increases, we say it accelerates. When its velocity decreases, we say it has negative acceleration, or decelerates. Average acceleration is change in velocity per unit of time:

$$\text{Average acceleration} = \frac{\text{Change in velocity}}{\text{Elapsed time}}. \tag{4.12}$$

Suppose I bow a string on a violin with starting velocity v_0 at time t_0 and end with final velocity v at time t. During the elapsed time interval $\Delta t = t - t_0$, the change in velocity is $\Delta v = v - v_0$. The average acceleration is:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0}. \qquad\qquad \textit{Average Acceleration} \quad (4.13)$$

The average acceleration $\bar{a}$ is a vector that points in the same direction as Δv.

We can determine the acceleration at a particular moment of time using the approach taken above for instantaneous velocity (4.11). We define instantaneous acceleration as

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}. \qquad\qquad \textit{Instantaneous Acceleration} \quad (4.14)$$

When the time interval Δt becomes extremely small so that it approaches zero in the limit, the average acceleration and the instantaneous acceleration become equal. Thus, instantaneous acceleration is the limiting case of the average acceleration. The ratio $\Delta v / \Delta t$ will *not* be infinitesimally small because it is a ratio of two small but nonzero values. If acceleration is constant, the acceleration will have the same value at any instant of time. In this book, when I say "acceleration," I mean instantaneous acceleration.

The SI unit of average acceleration is meters per second squared, or m/s^2. To see why, let us take an example. Suppose I accelerate my violin bow from a velocity of 0.1 m/s to a final velocity of 1 m/s over a time interval of 0.5 s. The average acceleration is:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0} = \frac{1.0\,\text{m/s} - 0.1\,\text{m/s}}{0.5\,\text{s} - 0\,\text{s}} = 1.8\,\text{m/s}^2. \tag{4.15}$$

4.10.1   Acceleration as the Bending of a Curve

Let's say that the car we filmed (figure 4.1) was accelerating. After filming it, we put the film in a projector and start viewing it. The projector shows us the frames in the order $i = 0, 1, 2, \ldots$, with elapsed time Δt between each one, so we view the motion at the same speed it was filmed.

As we watch the car accelerate away from the origin, suppose I suddenly stop the projector at an arbitrary frame i so that now we see the car frozen in time at some moment $t_i = i \cdot \Delta t$. Now we can determine the car's average acceleration from just this frame, the previous one, and the next one.