Multiplying Signals Convolves Their Spectra

Gareth Loy

In *Musimathics* V2 section 4.4.2 shows two proofs that multiplying signals convolves their spectra. Step 2 of that process, beginning on page 170, does not really provide enough information for the reader, and furthermore, Figure 4.13 has some misleading information. This document corrects the mistakes and fills in the missing information.

The definition of *f* and *g* shown below, printed in *Musimathics* V2 p. 171 Fig. 4.13, are very poorly trucated versions of sinusoids at 1 Hz and 3 Hz.
Here's what was printed in the book:

```
In[8]:= f = {0, 0.3, .6, .9, 1, .9, .8, .6, .18, -.18, -.6, -.8, -.9, -1, -.9, -.6, -.3};
        g = {0, .9, .8, -.18, -.9, -.6, .3, 1, .6, -.6, -1, -.3, .6, .9, .18, -.8, -.9};
```

Here is how *f* and *g* were originally generated:

```
In[10]:= f1 = Table[Sin[2 π n / 17], {n, 0, 16}];
         g1 = Table[Sin[6 π n / 17], {n, 0, 16}];
```

Here is what should have been printed in the book for these functions:

```
In[12]:= N[f1, 2]
         N[g1, 2]

Out[13]= {0, 0.90, 0.80, -0.18, -0.96, -0.67, 0.36, 1.0,
          0.53, -0.53, -1.0, -0.36, 0.67, 0.96, 0.18, -0.80, -0.90}

Out[12]= {0, 0.36, 0.67, 0.90, 1.0, 0.96, 0.80, 0.53,
          0.18, -0.18, -0.53, -0.80, -0.96, -1.0, -0.90, -0.67, -0.36}
```

But even these corrected functions are still inappropriate for what I was trying to show because they don't have enough precision.

Mathematica carries functions *f* and *g* with much more precision than two decimal digits, so we can use *Mathematica* to perform the calculations shown in Figure 4.13 to demonstrate that it is fundamentally correct.

We form the product of *f* and *g* :

```
In[14]:= h = f1 g1;
```

Here are the values that should have been shown for $f(n)\,g(n)$.

```
In[15]:= N[h, 2]

Out[15]= {0, 0.32, 0.54, -0.16, -0.96, -0.65, 0.29, 0.52,
          0.097, 0.097, 0.52, 0.29, -0.65, -0.96, -0.16, 0.54, 0.32}
```

Finally, take the Fourier transform of *h* and put it in rotated order (so that 0 Hz is in the center, negative frequencies are to the left of 0, positive frequencies are to the right).

In[16]:=
```
Chop[RotateRight[Abs[Fourier[h] / Sqrt[17]], 8]]
```

Out[16]=
```
{0, 0, 0, 0, 0.25, 0, 0.25, 0, 0, 0, 0.25, 0, 0.25, 0, 0, 0, 0}
```

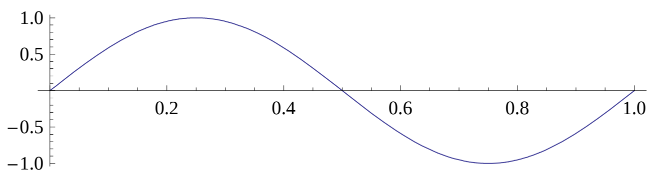
These values are exactly the ones shown for $H_2(k)$ in Figure 3.14, so the calculation is correct, but only if you use full precision, not the truncated precision shown in the book.

Generating the Graphical Functions

Here's the graphical functions shown in Figure 4.13.

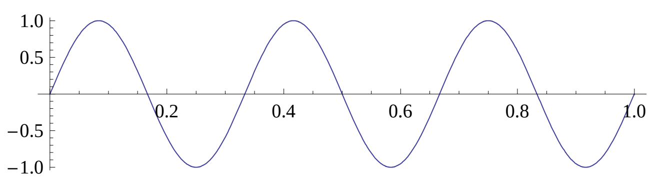
In[4]:=
```
Plot[Sin[2 π x], {x, 0, 1}, AspectRatio -> .25]
```

Out[4]=



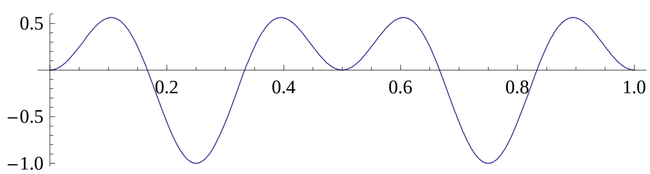
In[5]:=
```
Plot[Sin[6 π x], {x, 0, 1}, AspectRatio -> .25]
```

Out[5]=



In[6]:=
```
Plot[Sin[2 π x] Sin[6 π x], {x, 0, 1}, AspectRatio -> .25]
```

Out[6]=



In[17]:=
```
ListPlot[RotateRight[Abs[Fourier[h]] / Sqrt[17], 8], Filling -> Axis, AspectRatio -> .25]
```

Out[17]=